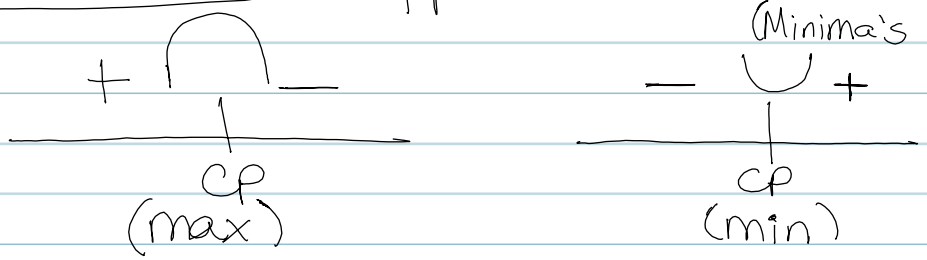
Section 5.2 Applications of Mins & Max's

(Minima's & maxima's)



Procedure:

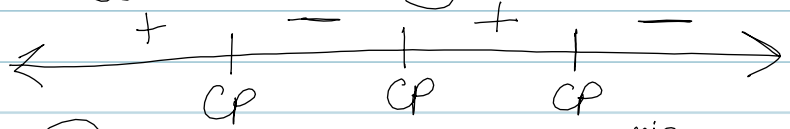
STEP 1) Determine Goal (G); what do we need to minimize or maximize?

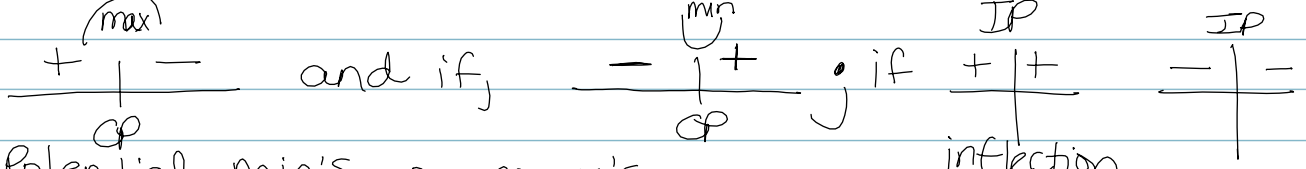
STEP 2) Set up the equation for your goal (G)

STEP 3) Find 1st derivative G'

STEP 4) Analyze G' by finding where G' = ∅ and where G' = undefined (Critical points) CP

STEP 5) Analyze G' using a # line





CP = Potential min's or max's

STEP 6) Find the information that was actually requested by plugging critical points (proper CP) into the original equation (from step # 2 above)

NOTE: The 2nd derivative being set to ∅ tells us when diminishing return has set in.

Ex #2 Maximize $P = xy$ when $x + 2y = 40$

STEP 1 Solve for one variable from equation

$x + 2y = 40$

, so $x = 40 - 2y$

Step 2 $P = (40 - 2y)y$ $\quad P = 40y - 2y^2$] Plug into $P = xy$ equation. Simplify

Step 3 Find derivative of $P = 40y - 2y^2$

, so $P' = 40 - 4y$

Step 4 Find $P' = \emptyset$ and $P' = \cancel{\text{undefined}}$

None b/c no denominator

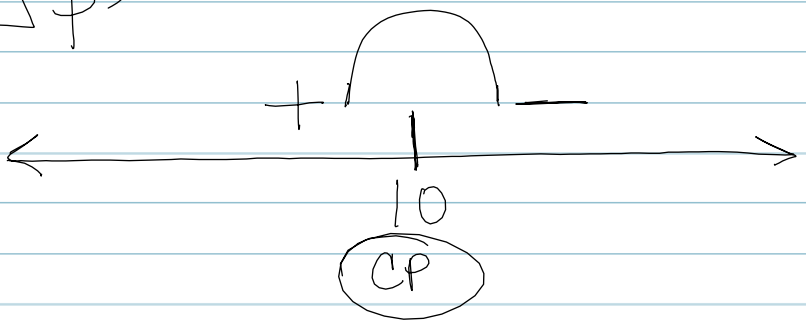
$40 - 4y = \emptyset$

$-40 \quad -40$

$\frac{-4y}{-4} = \frac{-40}{-4}$

$y = 10$

Step 5 P'



$P'(9) = 40 - 4(9) = +4$

$P'(11) = 40 - 4(11) = -4$

STEP 6 (P) is max when $x = ?$ and $y = 10$

Plug $y = 10$ into original to solve for (x), so

$x+2y=40$;

$x+2(10)=40$;

$x=20$

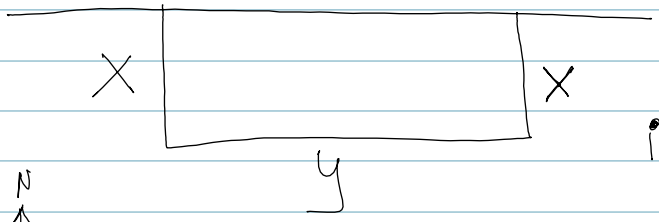
, so when $y=10$ and $x=20$ the

max P is $P=xy=(20)(10)=200$

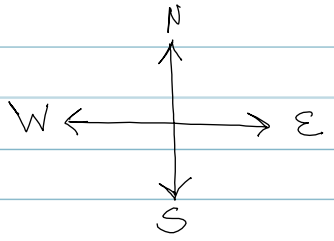
#18 Fences :

Step 1 GOAL: Largest (max) area of rectangular garden

Step 2 Draw picture



Equation of area of rectangle is $A=x \cdot y$



$y \rightarrow \$4/\text{foot}$

$x \rightarrow \$2/\text{foot}$

$\underset{x}{\text{Cost}} + \underset{y}{\text{Cost}} = \underset{\$80}{\text{Total budget}}$; $2x(\$2)+y(4)=80$

New equation: $4x+4y=80$
(Cleaned up!)

Factor out 4's = $x+y=20$

Solve for a variable ; $y=20-x$

Step 2 cont. Plug in $y = 20 - x$ into the Area equation $A = xy$

so $A = x(20-x)$

$A = 20x - x^2$

Step 3 Find $A' = 20 - 2x$

Step 4 Find $A' = \emptyset$

$20 - 2x = 0$

$+2x \quad +2x$

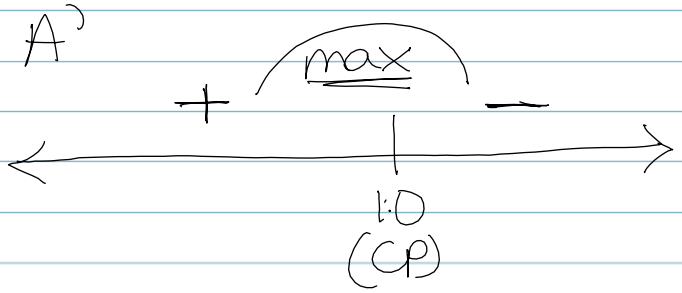
$\frac{20}{2} = \frac{2x}{2}$

$10 = x$ CP

Find $A' =$ ~~undefined~~

None b/c no denominator

Step 5 A'



$A'(9) = 20 - 2(9) = 2$ (+)

$A'(11) = 20 - 2(11) = -2$ (−)

Step 6 make sure we answer the right question.

GOAL: What are dimensions $l * w$ or $x * y$

* Use original equation of $y = 20 - x$ and solve for (y)

so $y = 20 - 10 = 10$ $y = 10$

Garden dimensions are $10 \text{ ft} \times 10 \text{ ft}$